

M.Sc. (Mathematics) (New CBCS Pattern) Semester-II  
**PSCMTH09 - Classical Mechanics**

P. Pages : 2

Time : Three Hours



**GUG/S/25/13749**

Max. Marks : 100

- Notes : 1. Solve all **five** questions.  
2. Each question carries equal marks.

**UNIT – I**

1. a) Find the shortest distance between two points in a plane. 10  
b) Derive the Lagrange's equations from Hamilton's Principle. 10

**OR**

- c) Prove that the generalized momentum conjugate to a cyclic coordinate is conserved. 10  
d) Show that a hoop rolls down the incline with one half the acceleration it would have slipping down a frictionless plane and the friction force of constrain is 10  
$$\lambda = \frac{Mg \sin \theta}{2}$$

**UNIT – II**

2. a) Explain the Routh's procedure. 10  
b) State and prove the Principle of least action. 10

**OR**

- c) State and prove relation between  $\delta$  and  $\Delta$ . 10  
d) If the constraint are independent of time for the equation. 10  
 $r_1 = r_1(q_1, q_2, \dots, q_n, t)$   
Do not involve time  $t$  explicitly then show that  
$$\Delta \int 2T dt = 0$$

**UNIT – III**

3. a) If  $F = F_1(q, Q, t)$  be the generating function of canonical transformation of 10  
 $P_1 = P_1(q, p, t), Q_1 = Q_1(q, p, t), i = 1, \dots, n$   
Then Prove that  
$$p_1 = \frac{\partial F_1}{\partial q_1}$$
$$P_1 = - \frac{\partial F_1}{\partial Q_1}$$
$$K = H + \frac{\partial F_1}{\partial t}$$
  
Where  $K$  is the changed Hamiltonian in new variable  $P, Q$ .

- b) Prove that The Poisson bracket of u, v is- 10

$$[u, v] = \frac{\partial u \partial v}{\partial q_i \partial p_i} - \frac{\partial u \partial v}{\partial p_i \partial q_i}$$

Invariant under all transformation of coordinate.

**OR**

- c) Prove that- 10

i)  $[q_i, p_j] = 0$

ii)  $[p_i, p_j] = 0,$

iii)  $[q_i, p_j] = \delta_{ij}$

- d) Prove that- 10

i)  $[p_i, H] = \dot{p}_i$

ii)  $[q_i, H] = \dot{q}_i$

#### UNIT – IV

4. a) Define Infinitesimal canonical transformations. 10

- b) Prove that The Hamiltonian is the generator of the system motion with time. 10

**OR**

- c) State and prove the Liouville's theorem. 10

- d) Discuss the Symmetry groups of mechanical systems. 10

5. a) Define the Hamilton's Principle. 5

- b) Define Legendre transformations. 5

- c) Prove that The Poisson bracket of two constant of motion is itself constant of motion. 5

- d) If u does not contain time t explicitly, then prove that 5

$$\frac{du}{dt} = [u, H]$$

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